Proceedings of the
Symposium on Christoph Clavius (1538–1612)
July 21, 2005
University of Notre Dame

Edited by Dennis Snow

Contents

Preface

The *Symposium on Christoph Clavius (1538–1612)* was held at the University of Notre Dame on Thursday, July 21, 2005, sponsored by:

- The Clavius Mathematical Research Group, an international association of Catholic mathematicians founded in 1963 by Andrew Whitman and Lawrence Conlon that meets every year in July for seminars, reflection, and research. In recent years the host institutions have been the Institute for Advanced Study, the Institut des Hautes Etudes Scientifiques (Bures-sur-Yvette, France), Boston College, the University of Notre Dame, Fairfield University, and the College of the Holy Cross.
- The Library of the University of Notre Dame.
- The Department of Mathematics of the University of Notre Dame.

The occasion of the symposium was the inauguration of the website containing the collected works of Clavius, the *Opera Mathematica*,

`http://mathematics.library.nd.edu/clavius`

The symposium and the website are dedicated to the memory of

The Reverend Joseph MacDonnell, S.J.
May 4, 1929 – June 14, 2005

Fr. MacDonnell was keenly interested in the project to make Clavius' works available online throughout its development and provided financial support to sustain it.

There are similarities in appearance between Clavius as shown in the illustration on the title page and the above photograph of Fr. MacDonnell. Both were mathematicians, yet both shared a deep respect for physical observations. Fr. MacDonnell liked to show people things, especially his string models of ruled surfaces.

The Atrium in the Bannow Science Hall, Fairfield University, dedicated to Fr. MacDonnell and containing some of his string models. Fr. MacDonnell had been a member of the mathematics faculty at Fairfield University since 1969.

The following description of Christoph Clavius was given by author and commentator David Ewing Duncan (*On Time: Today's calendar owes a debt to a Jesuit*, The Company Magazine, March 3, 2000). It is surprising how aptly the description fits Fr. MacDonnell as well.

> Christopher Clavius helped revolutionize the calendar, yet little exists to flesh out who he really was. In a portrait of Clavius rendered in 1606 he is dressed in a simple Jesuit robe and a four-cornered hat. A portly, satisfied-looking man with a pudgy, bearded face, he looks sympathetic, even kind–the sort of scholar

who is serious but never stuffy, smart but not precocious; one that students are fond of, and one that politicians and prelates feel comfortable assigning to commissions.

Symposium Schedule

9:15 Hayes-Healey 127
Dedication to the Rev. Joseph MacDonnell, S.J.

9:30 Hayes-Healey 127
An overview of Clavius' life and work
Paul Schweitzer, S.J.
Departamento de Matematica
Pontificia Universidade Catolica do Rio de Janeiro

11:00 Hayes-Healey 127
Clavius' contribution to the Gregorian reform of the calendar
Dennis Snow
Department of Mathematics
University of Notre Dame

2:00 Hayes-Healey 127
What's so great about mathematics? Some Renaissance responses
Robert Goulding
Program in the History and Philosophy of Science
University of Notre Dame

3:00 Hurley 257 (Commons Room)
Reception

4:00 Hayes-Healey 127
The Clavius Project: Putting Clavius' Opera Mathematica *online*
Parker Ladwig
Mathematics Library
University of Notre Dame

5:30 The Chapel in the Coleman-Morse Center
Memorial Mass for the Rev. John MacDonnell, S.J., and the Rev. Joseph MacDonnell, S.J.

CHAPTER 1

An overview of the life and work of Christopher Clavius

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If Fr. Joseph MacDonnell, S.J. were still alive today, he would certainly be the one to give this lecture on the life and work of Christopher Clavius. In fact, virtually all of the references to Clavius' work that I have found, aside from direct consultation of Clavius' own publications, are mentioned in MacDonnell's excellent little book *Jesuit Geometers*. Just as today's symposium is dedicated to the memory of Joe MacDonnell, I would like to dedicate this lecture to him, and ask his indulgence since he would certainly have done a better job.

I shall treat the following topics:

- Clavius' life
- Clavius at the Roman College
- Clavius' publications: Commentaries on Euclid and Sacro Bosco and his textbooks
- Clavius' contribution to the development of mathematical notation
- Clavius as astronomer and scientist
- Clavius as mathematician

and then give a brief Conclusion.

A brief outline of Clavius' life

Christopher Clavius was born on 25 March 1538 in Bamberg, Franconia, in Germany. According to the Jesuit archives in Rome, Clavius was in Rome in February 1555 and entered the novitiate there on 12 April 1555, shortly after his seventeenth birthday.

In 1556, he was sent to the newly instituted Jesuit College at Coimbra, founded in 1555, to study at the University of Coimbra. In August 1560, while he was still at Coimbra, he observed the total eclipse of the Sun (*Sphaera*, 1511, p.265), and this experience was instrumental in leading him to choose a career in science and mathematics. He must have learned mathematics and science from professors there, though it seems that he also learned much by his own study.

Still in 1560, Clavius returned to Rome to study theology. In 1563 he began teaching as a professor in the Roman College and was ordained in 1564. He taught at the Roman College for many years and was a professor there for 45 years. After an illness of a few months, Clavius died in Rome, at the Roman College, on February 6, 1612.

A contemporary of Clavius, Bernardino Baldi, describes Clavius when he was about fifty years old:

> He is a man untiring in his studies and is of a constitution so robust that he can endure comfortably the long evenings and efforts of scholarship. In stature he is well proportioned and strong. He has an agreeable face with a masculine blush, and his hair is mixed in black and white. He speaks Italian very well, speaks Latin elegantly, and understands Greek. But as important as all these things, his disposition is such that he is pleasant with all those who converse with him. At the time of this writing he is in his 50th year, and we should pray that that life is prolonged so that the world may continue to receive those fruits that intellects as cultured and fertile as his are accustomed to produce. (cited by James M. Lattis, *Between Copernicus and Galileo—Christoph Clavius and the Collapse of Ptolemaic Cosmology*, p.22, from Guido Zaccagni, *Bernardino Baldi nella vita e nella opere*, 2nd ed., Pistoia: Soc. An. Tipo-Litografia Toscana, 1908, pp.344–345)

Lattis comments that a small number of musical compositions by Clavius have survived (p.23) and also mentions that Clavius traveled around Italy and reportedly visited Spain and Germany.

Clavius at the *Collegio Romano*

Clavius had a long career as professor at the *Collegio Romano*, the Jesuit university in Rome, now called the *Gregorian University*, which functioned as a model for Jesuit universities throughout Europe. Extant documents show that he taught there from 1564 to 1595, but his tenure was even lengthier. I believe that he was instrumental in bringing about the involvement and excellence of the first and second generation of Jesuits in mathematics and science. Nevertheless, I have not found the specific evidence that I have expected to find that would prove that statement conclusively. In any case, the explosion of mathematical and scientific talent that irradiated from the *Collegio Romano*, as related in MacDonnell's book *Jesuit Scientists*, was an extraordinary event in the history of science.

A comment by Clavius' Jesuit successor Christopher Grienberger, when he was trying to understand Gregory of St. Vincent's effort to square the circle (a challenging problem of that time, since the impossibility of solving it had not yet been shown), shows the reverence and respect that Clavius' contemporaries at the *Collegio Romano* had towards him. Grienberger lamented, "If only Clavius were alive now! How I miss his counsel." (J. MacDonnell, op. cit., p.8)

An example of Clavius' work in spreading scientific knowledge is his interaction with the young professor Galileo at Pisa. Galileo wrote to Clavius, an elder statesman of science and mathematics whom he deeply respected, asking for copies of course notes and books from the *Collegio Romano.* William Wallace, in his interesting works *Galileo's Early Notebooks: The Physical Questions* (University of Notre Dame Press, 1977) *Galileo and his Sources* (Princeton University Press, 1984), shows that about 90% of three notebooks of Galileo from that early period (1589–1591) were taken from notes of professors at the *Collegio Romano*, mostly Jesuits, including Franciscus Toletus, Ludovicus Carbone, Paulus Valla, Ioannes Lorinus, Mutius Vitelleschi, Ludovicus Rugerius, Robertus Jones, and Andreas Eudaemon-Ioannis. These notebooks were mistakenly dated about ten years earlier by Antonio Favaro, the editor of Galileo's complete works, and consequently had not received the attention they deserved before Wallace's revealing studies. It is very likely that it was Clavius, in response to Galileo's request addressed to him, who obtained these class notes for him.

It was Clavius who instructed Robert Bellarmine about Galileo's discoveries in astronomy. Bellarmine showed a clear understanding of the value of the discoveries and their relationship to the question of the interpretation of the Old Testament. Unfortunately, when Galileo confronted the Inquisition in 1616 and in 1633, Clavius was no longer alive. It is interesting to speculate that, had Clavius been present on these two occasions, the egregious error of Church authorities in condemning Galileo might have been avoided.

Clavius' publications: Commentary on Euclid, Sphaeram, textbooks

Clavius wrote various influential books. His commentary on the *Elements of Euclid*, which went through many editions (*Euclidis Elementorum Libri XV*, with editions in 1574, 1591, 1605, 1612, 1627, and 1654), was a compendium of what was known in geometry in his day, and was constantly being updated with the latest discoveries in geometry. His commentary on the *Sphere* of John Hollywood, *In sphaeram Joannis de Sacro Bosco commentarius* (Rome, 1581; 3rd ed., Venice 1601, cf. Clavius' *Works*), a treatise on the geometry of the sphere and the astronomy of the time, was extensively used and referenced by later writers. He wrote another influential book on the *Astrolabe* (*Works*, vol.3).

These three works of Clavius were complemented by a number of textbooks, on algebra, arithmetic, and practical geometry, which were widely used in all of Europe and even in other continents for many decades. These textbooks—*Geometria Practica*, *Arithmetica Practica*, and *Algebra*—are published in vol.2 of Clavius' *Works*.

These and other works of Clavius are available, in the original Latin and some in English translation, online at `http://mathematics.library.nd.edu/clavius` here at Notre Dame. The contents of this 1611 edition of Clavius' *Works* are as follows:

Volume 1: Table of Contents for *Opera Mathematica*, *Prolegomena to the mathematical disciplines*, *Commentary on Euclid*, *Commentary on Theodosius*, *On Secant and Tangent Lines* (with a 7-place table of sines), *Plane Triangles*, *Spherical Triangles*.

Volume 2: *Practical Geometry*, *Practical Arithmetic*, and *Algebra*.

Volume 3: *Commentary on John Hollywood's Spheres*, and *Astrolabe*.

Volume 4: *Gnomonics*, *Construction and Use of the Sun Dial*, and *New Description of the Sun Dial*.

Volume 5: *Roman Calendar of Gregory XIII*, *Apology against Michael Maestlin*, and some appendices to the *Apology*.

Concerning the influence of Clavius' books, MacDonnell cites the following laudatory remarks of the historian Moritz Cantor:

> The large number of editions which were needed to satisfy the demand for his Euclid shows the high reputation which the work achieved. Seldom has so high a reputation been so well earned... Clavius shows an acute critical faculty. He detected and exposed old errors. There is no difficulty that he attempted to evade. ... This work of Clavius is indispensable even today for historical research. (Moritz Cantor, *Vorlesungen über Geschichte der Mathematik*, Leipzig: 1900–1908, vol.2, p.512, cited in *Jesuit Geometers*, p.7)

MacDonnell cites another historian, Abraham Kaestner, who refers to Clavius' *Geometria Practica* (1604) as "a model textbook of practical geometry, perfect for its time." (MacDonnell, p.7). Leibniz was led to an interest in mathematics by reading books by Jesuits such as Clavius and Gregory of St. Vincent (J.M. Child, *The Early Mathematical Manuscripts of Leibniz*, Chicago: Open Court, 1920, cited in MacDonnell, p.7), and writes that he learned algebra from Clavius' textbook.

William Wallace comments on the positive attitude toward mathematics which Clavius adopted and defended, for example, in the *Prolegomena* to his second edition of Euclid's *Elements*, published in 1589. This edition also contained "substantial additions from Archimedes, Apollonius, Ptolemy, and others in order to connect their works with Euclid's theorems" (Wallace, *Galileo and his Sources*, pp.137–138). Wallace comments that Clavius gave first place among the sciences to mathematics, because it studies things apart from being immersed in sensible matter and attains certitude through its demonstrations, unlike other disciplines where there were continual disputes and arguments that leave the student in doubt (ibid. 138–139).

Clavius' edition of Euclid was translated into Chinese by his student Matteo Ricci, and thus his influence reached distant points of the globe. His work was also favorably commented on in various articles in the Philosophical Transactions of the Royal Society of London, despite the intense religious persecution of Jesuits in England at the time.

Clavius' contribution to the development of modern mathematical notation

F. Cajori's encyclopedic work, the two-volume *A History of Mathematical Notation*, discusses the gradual development of several modern mathematical notations in the course of the sixteenth and early seventeenth centuries. He cites an article of Jekuthiel Ginsburg (*The early history of the decimal point*, Amer. Math. Monthly 35 (1928), 347–349) showing that Clavius apparently was the first to use a decimal point in the 1593 edition of his book *Astrolabe*, in which Clavius writes 46.5 to represent 46 plus five tenths. Clavius also uses the dot to separate integers, and not exclusively as a decimal separatrix, but Cajori concludes, "Nevertheless, Clavius unquestionably deserves a place in the history of the introduction of the dot as a decimal separatrix" (Cajori, vol.1, p.322).

Cajori discusses Clavius' notations at some length (vol.1, pp.151–154). He affirms that "Clavius is one of the very first to use round parentheses to express aggregation." He notes that it was Clavius who, in his *Algebra* of 1608, introduced the notations $+$ and $-$ for addition and subtraction, already used in Germany by Michael Stifel, to Italy. Cajori quotes the text in which Clavius explicitly explains his preference for these two signs, instead of the letters P and M commonly used at the time for plus and minus, to avoid confusion with the letters used to represent numbers (vol.1, p.151). On page 153 Cajori reproduces the page 159 from the 1608 edition of Clavius' *Algebra*, where he writes products of sums of integer multiples of square roots much as we would write the multiplication today, the only difference being that the square root was represented by the radical sign followed by a script letter (probably r), and that there is an accompanying text in Latin explaining each step. For an unknown quantity, Clavius used a cursive letter (apparently x) and for additional unknowns, he used $1A$, $1B$, etc. He writes $3x + 4A$, $4B - 3A$ for $3x + 4y$, $4z - 3y$. Clavius also used the modern notation for fractions, one integer over another with an underscore between, in the 1601 edition of his *Arithmetic*, although he had omitted the underscore after the first fraction in a product of several fractions in an earlier edition.

In conclusion, we can say that Clavius was one of the first to use the mathematical notations which have become standard today, and he was apparently the very first to use the decimal point. When we consider that his textbooks were widely used throughout Europe for many years, we must recognize unquestionably that he made an important contribution to the evolution of mathematical notation.

Clavius as astronomer and scientist

Clavius made contributions to astronomy, science, and technology. His contribution to the calendar reform is well known and will be treated by Prof. Dennis Snow in the next lecture, so I am content merely to mention it here. I shall mention only a few examples of the impact of his work in science.

Clavius' observation of the solar eclipse of 9 April 1567 in Rome was important because he described it as an annular eclipse, one in which a circular ring of the solar disk appeared around the dark disk of the Moon. It was cited by recent astronomers to support the contention that the size of the Sun has shrunk over the last centuries (Lattis, op. cit., p.18, n.66).

In 1611 he published a report showing that most of Galileo's astronomical observations were correct. Clavius' enormous scientific authority was thus significant in obtaining acceptance for Galileo's discoveries, which went counter to the Aristotelian positions that were strongly held in academic circles at the time.

His commentary on the *Sphere* of Sacro Bosco, mentioned above, gave detailed instructions on how to make astronomical calculations using spherical geometry. It was a basic instrument in the development of astronomy at the time.

Clavius introduced a method similar to the later Vernier scale for reading precise measurements. His method permitted more precise astronomical observations.

Clavius as mathematician

We have seen that the work of Clavius in systematizing mathematical knowledge of his time was a significant contribution to the advancement of mathematics. In particular, the many editions of his commentary on Euclid, a compendium of what was known then in geometry, earned for him the title "the Euclid of the 16th century," by which he was frequently referred to. In addition to this work of organization and divulgation, however, Clavius made original contributions to mathematics.

Clavius published a carefully checked table of sines with values to seven decimal places for every angle, minute by minute, from 0 to 90 degrees. It was based on an earlier table published by John Regiomontanus, but Clavius made numerous corrections. He also explained, in the accompanying text, how interpolation would give values of the sine of angles with fractional minutes to an equal degree of precision, and he gave many examples to illustrate its use.

Clavius invented a forerunner of logarithms, a useful numerical method for simplifying the calculation of the product of large numbers. The method reduces a multiplication of two large numbers to a small number of sums and differences, using cosines, which of course can be read directly from his table of sines. This method, which he called *prosthaphaeresis* (from Greek roots that mean adding and subtracting), was used by Tycho Brahe in his astronomical calculations. A writer of the period commented that it doubled the life of astronomers, since it permitted them to do calculations so much more rapidly. Clavius' method is based on the trigonometric formula

$$\cos a \cos b = \frac{1}{2}[\cos(a+b) + \cos(a-b)]$$

The multiplication on the left side is effected by calculating three sums or differences and four table look-ups. The method works as follows. Given two large numbers x and y, move the decimal points to make them lie between 0.1 and 1. Then use the sine table to find the angles a and b whose cosines are x and y, respectively. One calculates $a+b$ and $a-b$, looks up their cosines in the table, and then their average is the product xy, up to multiplication by an obvious power of 10.

This *proto-logarithm* using cosines had limited influence in the history of mathematics, since Napier discovered logarithms and published a paper on them in 1614, and, shortly afterwards (in 1619), he published his table of logarithms. Thus Clavius' prosthaphaeresis was superseded in a couple of decades by the simpler method of calculation using logarithms. Nevertheless, Clavius' method seems to be first numerical method in history to simplify the calculation of products of large numbers.

In mathematical logic, a discipline that was little developed in the time of Clavius, he made interesting use of the logical argument

$$(\sim p \Rightarrow p) \Rightarrow p$$

If from the negation of a proposition p one can prove p, then it follows that p is true. This is a trivial observation of today's mathematical logic, but at Clavius' time, when Aristotelian logic was the standard, it was a new development.

Clavius revived interest in the question of the status of Euclid's Fifth Postulate about parallel lines, which states that through a point outside a given straight line exactly one straight line can be drawn parallel to the given line (i.e., such that it will never intersect the given straight line, however far they may be extended). Interest in its relationship to the other axioms and postulates of Euclid had been largely dormant for centuries. Clavius returned to the question of whether this postulate could be proven from Euclid's other axioms and postulates. He showed that the parallel postulate was equivalent to the assertion that points in the plane equidistant from a straight line on one side also form a straight line.

Clavius proved a theorem about the ratio of the angles of the triangle formed by a side of a regular polygon of $(2n+1)$ sides and the opposite vertex. He showed that the ratio of the angles is n. This fact was used by Gauss in his well-known geometric construction of the regular polygon with 17 sides, which also shows that the seventeen complex roots of the equation $x^{17} - 1 = 0$ can be constructed by ruler and compass.

Conclusion: The impact of Clavius on science and mathematics

The preceding reflections clearly show the importance of Clavius in many dimensions of the development of science and mathematics. I was surprised to discover that recent editions of the *Encyclopedia Britannica* and the *Encyclopedia Universalis* do not even have an entry for Clavius. I hope that the online availability of Clavius works here at Notre Dame, and future expansion of this valuable site, especially by the inclusion of translations and works about Clavius, will make him better known and appreciated. Let us hope that the importance of Clavius in the development of Renaissance and early modern science and mathematics will soon come to be fully recognized.

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CHAPTER 2

Clavius' contribution to the Gregorian reform of the calendar

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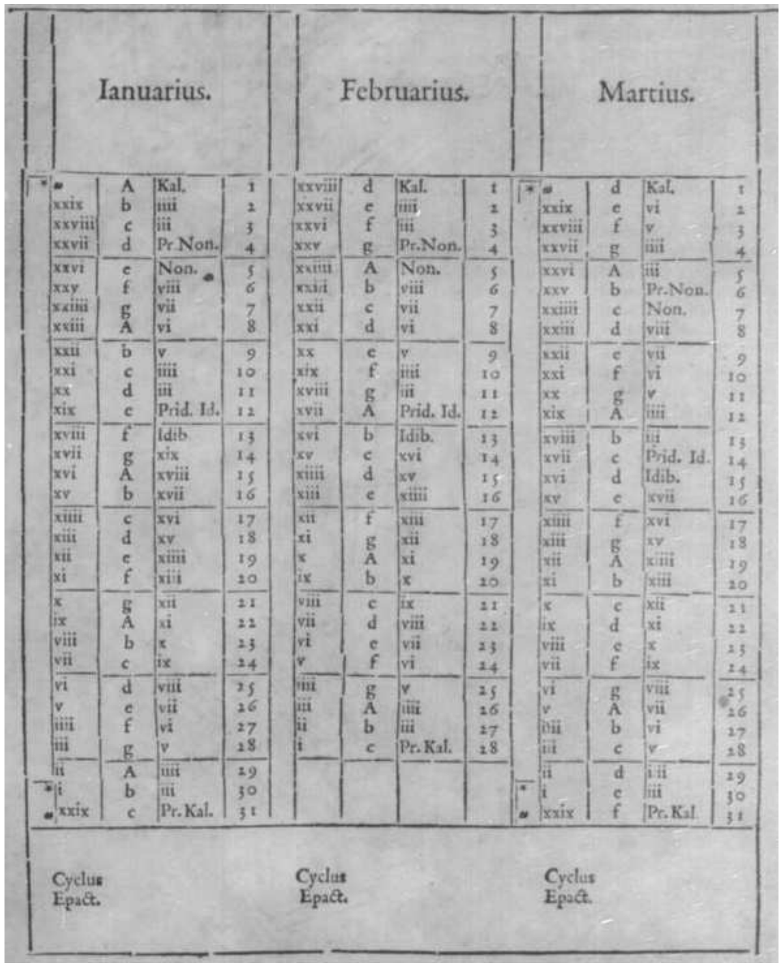
In 1582 Pope Gregory XIII issued a papal bull, *Inter gravissimas*, that made the following changes to the calender:

- Changed the rule for leap years.
- Established a rule for deciding the date of Easter.
- Deleted 10 days from October 1582 to restore the vernal equinox to March 21.

The bull was the culmination of the work of an international commission established by Gregory XIII in the early 1570's. Out of the nine commission members, all but one was a member of the clergy—he was a lawyer witnessing the Arabic signature of a patriarch from the east—and only two were scientists, Ignatio Danti and Christoph Clavius. The composition of the commission is a clear indication that the primary motivation for the reform was not to fix the problem of the months drifting out of season as is popularly assumed, but it was rather to solve the problem of dating the feast of Easter in a uniform and acceptable manner. The plan adopted was actually proposed some time earlier by Luigi Giglio. Antonio Giglio was on the commission representing his deceased brother. The main contribution of Clavius was his wisdom in ensuring that the changes were based on average astronomical observations independent of any competing and, at the time, controversial theories, and his zeal and influence in promoting the adoption of the new calendar throughout Europe.

A brief history of the calendar before the Gregorian reform

A calendar is simply a way of keeping track of days. The word calendar comes from the Roman word for the first day of a month. Our modern calendar has a mixed heritage. The division of an hour into 60 minutes and a minute into 60 seconds was adopted from the Mesopotamians' sexagesimal system. The division of a day into 24 hours was adopted from the Egyptians. The seven-day week was passed down from the ancient Near East and appears already in Genesis. The Greeks introduced the convention for naming the days after the planets, and the Romans contributed most of the rest of our civil calendar, including the names of the twelve months and the number of days they contain.



Ianuarius.				Februarius.				Martius.			
*	A	Kal.	1	xxviii	d	Kal.	1	*	d	Kal.	1
xxix	b	iiii	2	xxvii	e	iiii	2	xxix	e	vi	2
xxviii	c	iii	3	xxvi	f	iii	3	xxviii	f	v	3
xxvii	d	Pr.Non.	4	xxv	g	Pr.Non.	4	xxvii	g	iiii	4
xxvi	e	Non.	5	xxiiii	A	Non.	5	xxvi	A	iii	5
xxv	f	viii	6	xxiii	b	viii	6	xxv	b	Pr.Non.	6
xxiiii	g	vii	7	xxii	c	vii	7	xxiiii	c	Non.	7
xxiii	A	vi	8	xxi	d	vi	8	xxiii	d	viii	8
xxii	b	v	9	xx	e	v	9	xxii	e	vii	9
xxi	c	iiii	10	xix	f	iiii	10	xxi	f	vi	10
xx	d	iii	11	xviii	g	iii	11	xx	g	v	11
xix	e	Prid. Id.	12	xvii	A	Prid. Id.	12	xix	A	iiii	12
xviii	f	Idib	13	xvi	b	Idib.	13	xviii	b	iii	13
xvii	g	xix	14	xv	c	xvi	14	xvii	c	Prid. Id.	14
xvi	A	xviii	15	xiiii	d	xv	15	xvi	d	Idib.	15
xv	b	xvii	16	xiii	e	xiiii	16	xv	e	xvii	16
xiiii	c	xvi	17	xii	f	xiii	17	xiiii	f	xvi	17
xiii	d	xv	18	xi	g	xii	18	xiii	g	xv	18
xii	e	xiiii	19	x	A	xi	19	xii	A	xiiii	19
xi	f	xiii	20	ix	b	x	20	xi	b	xiii	20
x	g	xii	21	viii	c	ix	21	x	c	xii	21
ix	A	xi	22	vii	d	viii	22	ix	d	xi	22
viii	b	x	23	vi	e	vii	23	viii	e	x	23
vii	c	ix	24	v	f	vi	24	vii	f	ix	24
vi	d	viii	25	iiii	g	v	25	vi	g	viii	25
v	e	vii	26	iii	A	iiii	26	v	A	vii	26
iiii	f	vi	27	ii	b	iii	27	iiii	b	vi	27
iii	g	v	28	i	c	Pr. Kal.	28	iii	c	v	28
ii	A	iiii	29					ii	d	iiii	29
i	b	iii	30					i	e	iii	30
xxix	c	Pr. Kal.	31					xxix	f	Pr. Kal	31
Cyclus Epact.				Cyclus Epact.				Cyclus Epact.			

Figure 1. The first three months of the calendar taken from Clavius' Opera Mathematica, Vol.5, p.5. *For each month the first column counts down the days to a new Moon, the second column gives the days of the week (labeled Abcdefg), the third column gives the Roman method for reckoning the days of the month, and the fourth column gives the day of the month.*

The Roman Calendar. In ancient Rome, there were only ten months, the first four named after familiar gods: Martius (Mars), Aprilis (Aphrodite/Venus), Maius (Maia), Junius (Juno), Quintilis (fifth), Sextilis (sixth), Septembris (seventh), Octobris (eighth), Novembris (ninth), Decembris (tenth). The total number of days in these months amounted to 304. It is uncertain how extra days or months were added. Around 700 BC, two months were added between Decembris and Martius: Februarius, named after Februa, a purification festival, and Januarius, named after Janus, the double-faced god, one face looking forward, the other back. Janus was regarded as the god of all beginnings.

In 452 BC, Februarius and Januarius switched places. The number of days of the month were regulated to alternate between 29 and 30. An extra day was added—for good luck—for a total of 355. To make up the yearly deficit, an extra month, called Intercalaris or Mercdonius (from the Latin *merces*, wages), was sometimes added between Februarius 23 and 24! The decision was made by a pontifex, a Roman religious leader, often for political reasons.

In 45 BC Julius Caesar tried to bring some order to the chaotic state of the calendar. The equinox was restored to March 25 after having been displaced over time in the civil calendar by three months. The start of the year was moved from March to January 1. The lengths of the months were fixed to alternate between 31 days for odd months and 30 days for even months, with Februarius having 29 or 30 days. A day was added between Feb 23 and 24 in a *leap year* which occurred every fourth year. During this period the month Quintilis and Sextilis were renamed to Julius and Augustus, respectively, to honor the two Caesars. There is an old story, perhaps going back to the 14th century, that a day was taken from Februarius and given to Augustus so that Augustus Caesar's month would have as many days as Julius Casar's month, but this claim appears to have no historical basis. With the adoption of the leap year rule, the average length of the year in the Julian calendar was 365.25 days. It was known to Hipparchus around 125 BC, that this value is too long. The actual error is about 11 minutes, 14 seconds per year, enough to cause the vernal (spring) equinox to fall back 1 day every 128 years. By the Council of Nicaea 325AD the vernal equinox fell on March 21 (4 days off) It is interesting to note that the scientific definition of a light year is defined using the average Julian year length of 365.25 days.

The Seven-day Week. The Greeks divided a month into three 10-day periods. The Romans marked each month with three special days: *Kalendae*, the first day of the month, *Idus*, the 13th or 15th day of the month, and *Nonae*, the 9th day before Idus, and reckoned dates by the number of days before and after these special days.

The seven-day week is very ancient and has uncertain origins in the Near East. In the Hellenistic period, 300–100 BC, the names of days were taken from the planet that ruled them. The scheme was derived as follows. The seven wandering heavenly bodies were commonly ordered by the speed as which they moved in relation to the fixed stars: Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon, with Saturn the slowest and the Moon the fastest. Each hour of each day was assigned to a planet in rotating order. The first hour of the first day was given to the Sun, so the first day was named Sunday. The second hour of the first day was given to Jupiter, the third hour to Mars, etc., and the 24th hour to Mercury. So the first hour of the second day was given to the Moon and the second day was named *Moon* day or Monday. The pattern continues with the third day named after Mars, the fourth after Mercury, the fifth after Jupiter, the sixth after Venus, and the seventh after Saturn. Old Norse equivalents were substituted for some of the Roman gods: *Tiw* for Mars, *Woden* for Mercury, *Thor* for Jupiter, and *Frigg* for Venus, giving the English names Tuesday, Wednesday, and Thursday, and Friday. There is no record that the seven-day week has ever been interrupted, possibly since before the days of Moses ca. 1400 BC. It is a work cycle that has proven over millennia to be well-adapted to human endurance.

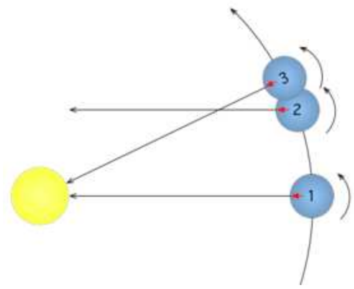


Figure 2. The Sidereal Day. *At time 1, the Sun and a certain distant star are both overhead. At time 2, the planet has rotated* $360°$ *and the distant star is overhead again* ($1 \to 2$ = *one sidereal day). But it is not until a little later, at time 3, that the Sun is overhead again* ($1 \to 3$ = *one solar day).*

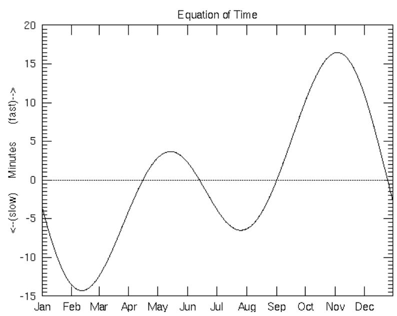
Problems with the calendar

The fundamental problem for all calendars is the incommensurate periods of the Sun, Moon, and rotation of the Earth. The slightly elliptical shape of the Earth's orbit around the Sun and the slow precession of the Earth's axis of rotation also present certain difficulties in giving precise definitions to the basic notions of a day, month and year.

What is a day? In seeking a scientific way to describe a day, one may be lead to declare a day as the length of time for the Earth to complete one $360°$ rotation about its axis. This definition is the same as the length of time it takes for a given star to cross the meridian on successive nights, and so is called a *sidereal day* (from the Latin *sidus*, star). Another way to define a day, called the *solar day* is the length of time it takes the Sun to cross the meridian on successive days.

It may be surprising to learn that a sidereal day is shorter than a solar day by about 3 minutes and 56 seconds. The reason is illustrated in Figure 2. The rotation of the Earth makes the Sun appear to move from east to west in the sky. However, the motion of the Earth in its orbit has the opposite effect: the Sun moves back slightly to the east every day. The combined effect is that after rotating $360°$, the apparent motion of the Sun will be just short of a complete circle around the Earth. In the figure, it can be seen that the Earth has to rotate a little more than $360°$ for the Sun to be directly overhead again. Over the course of a year, the total discrepancies between a sidereal and solar day amounts to one complete rotation of the Earth, so the number of sidereal days in a year, 366^+, is exactly one more than the number of solar days in a year, 365^+.

It may be even more surprising to learn that while the sidereal day is nearly constant, the solar day is not. There are two components that effect the length of the solar day: the tilt of the the Earth with respect to its orbital plane and the varying speed of the Earth in its slightly elliptical orbit. The Earth's tilt of $23.439°$ gives an apparent north/south component to the Sun's motion against the background stars. A larger north/south component is greatest at the equinoxes, resulting in a shorter eastward component and lengthening the day. The eastward component is greatest at the solstices where the north/south component is zero, shortening the day. The length of the solar day is further complicated by the fact that the



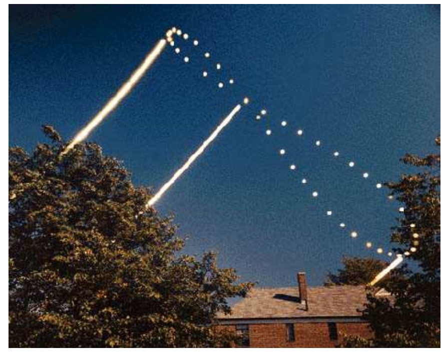


Figure 3. The Equation of Time. *The sum of the two effects on the length of the solar day: the tilt of the Earth's axis of rotation and the Earth's varying speed in its elliptical orbit. The combined effect is dramatically illustrated by the photograph on the right showing the figure of an analemma. The multiple exposure was done by Dennis DiCicco* (© Sky & Telescope) *taking the same picture of the sky at 8:30 AM on the same day of the week for every week of the year during 1978–1979.*

Earth moves faster when it closest to the Sun (during our winters), lengthening the day, and slower when it is farthest from the Sun (in our summers), shortening the day. The sum of these two effects is called the *Equation of Time*, see Figure 3. In the middle months of the year the length of the day is quite close to 24 hours, but around September 15 the days are 20 seconds shorter and around Christmas the days are 20 seconds longer. The extremes in the accumulated differences occur at the beginning of November when the day is 16 minutes ahead of average solar time and mid-February when it is 14 minutes behind average solar time. Because of these variations, a solar day (and 24 hours) is actually defined to be the *average* length of all the days in a year.

What is a year? As with measuring days, there is some difficulty giving a precise definition of a year. A *sidereal year* is the time required for one complete revolution of the Earth around the Sun, that is, the orbital period of the Earth. Its length is 365.3564 days.

A *tropical year* is the time it takes for the Sun, as viewed from the Earth, to re-appear in the same spot in the sky against the background stars. The effects that change the length of the day do not affect the length of the tropical year. However, the length of the tropical year depends on the time of year when the measurement is begun and ended. The difference, which seems mathematically impossible, is caused by the gradual drift in the position of the Earth's axis of rotation which is moving in a large circle that takes 25,800 years to complete. This *precession* of the Earth's axis is caused by the Sun's gravity tugging on the equitorial bulge of the rotating Earth and causes an apparent gradual shift in the position of all the stars and the Sun to the west. To see the effect on the length of the tropical year consider making the measurement between two successive winter solstices versus two successive summer solstices. Due to precession the second solstice will appear to happen at a spot in the Earth's orbit before the point of where the first solstice occurred. The gap between these solstices represents a shortening of the year. The length of time of this gap is shorter at the winter solstices because the Earth is moving faster than in the summer. The difference is small, but does amount to about 20 minutes and 24 seconds.

The *mean tropical year* is the average of all the tropical years and is 365.24219 days. For comparison, a tropical year measured from the vernal equinox is 365.2424 days.

The Gregorian reform

Calculating the date of Easter, known as the *computus*, was of central importance in the church calendar. A simplified version of the rule is that Easter falls on the first Sunday after the first full Moon after the vernal equinox. The connection with the lunar cycle is due to the fact that in the Christian tradition the crucifixion took place on (or a day before) Passover, and Passover occurs on the 14th Day of the Hebrew month Nisan. In the Hebrew calendar a new month begins when the crescent Moon is seen after sunset; thus, the 14th day of Nisan is a full Moon. Since a lunar month lasts for a little over 29.5 days, the Hebrew months alternate between 29 and 30 days, and there are normally 12 lunar months in a year with 11 days left over. An extra month (sometimes two) is added every three years to keep the Hebrew calendar synchronized with the seasons, e.g., so that Nisan occurs in the spring. In ancient times the adjustments were based on rough observations like the state of vegetation. A complete solar-lunar cycle fairly closely approximates 19 years. By comparison, the Islamic calendar is also lunar, but no attempt is made to keep it synchronized with the solar calendar so feast days can occur any time during the year.

In the early church there were different traditions as to when Easter was celebrated. Some groups insisted on Easter being near the 14th of Nisan, others that it must be on a Sunday. In 325 AD the Council of Nicaea declared that all churches should observe Easter on a Sunday with the exact date being determined each year by the authorities in Alexandria using their expertise in astronomical observations. The council recognized that the Julian year was too long, causing the vernal equinox to fall back in the year. It also recognized that the current state of recorded observations was not sufficient to define an accurate computable algorithm for the date of Easter.

The problem of the drift of the vernal equinox was cited again in 730 AD by Venerable Bede, an English monk. By 1200 AD there were many complaints about the drift and many recommendations to restore it to March 21, the date of the vernal equinox in the year of the Council of Nicaea. In 1267 AD Roger Bacon proposed removing one day—i.e., not adding a leap day—every 125 years. Finally in the early 1570's Pope Gregory XIII established a commission to solve this problem. As mentioned in the introduction, the commission was essentially all clergy, and its main charge was to fix the date of Easter. The commission adopted the proposal of Luigi Giglio that had been circulating since 1577. The papal bull was issued in 1582.

The part of the reform that is of chief interest today is the new leap year rule: leap days are added to every year divisible by 4, but not to century years unless they were multiples of 400. Thus, 1600 and 2000 were leap years, but 1700, 1800 and 1900 were not. There have been only 3 days adjustment to the Julian calendar in the last 420 years. The average Gregorian year is

$$365 + 1/4 - 1/100 + 1/400 = 365.2425$$

days. Using modern values for the average tropical year, that is only 26.784 seconds too long, or 1 day every 3225 years. The new rule was praised then and now for its accuracy. However, there is some misconception that Clavius, one of only two scientists on the commission, must have made some extraordinary observations and calculations to determine the length of the year with this degree of accuracy. Precise observations were certainly available from Copernicus and others, but they are not really needed to figure out a good way to adjust the leap year rule. For example, in 1580 the vernal equinox fell on March 11 and in 325 AD it fell on March 21. So the calendar had fallen back 10 days in 1250 years, suggesting that one leap day be dropped from the old rule every 125 years or so, the value suggested by Roger Bacon. A more correct value would be one leap day every 128 years. The Gregorian reform removes one leap day every 133.3 years.

Establishing an algorithm for Easter was not so easy, and real mathematical and astronomical expertise was required. Clavius originally thought the date of the vernal equinox—therefore determining the date of Easter by the above mentioned rule—should be based on astronomical observation. Other members of the commission favored an algorithm that could be computed without reference to an astronomical authority and that would avoid the inherent difficulties in disseminating the correct date throughout the Catholic church. Clavius quickly conformed to their view. The commission's rule for Easter was based on an inaccurate version of the Hebrew calendar, declaring that the vernal equinox is always March 21, as it was during the Council of Nicaea. The commission then designed an algorithm that would compute the next full Moon, which may differ from a real full Moon by one or two days—and Easter is the following Sunday. This algorithm is still the one used by the Catholic church today.

The most controversial aspect of the reform was its abrupt removal of 10 days to bring the vernal equinox back to March 21: October 4, 1582 was followed by October 15, 1582. The days of the week were not interrupted. This gap was the cause of much protest and outright rejection of the Gregorian calendar, especially in Protestant countries. England and its colonies did not adopt the reform until 1752. George Washington's birthday occurred on February 11, 1732 in the English calendar, but that date became the more familiar February 22 after the change. Many adoptions occurred quite late: Alaska in 1867, Russia in 1918, Greece in 1923, Turkey in 1927, and China in 1928. Today, the Gregorian calendar is the standard civil calendar used throughout the world, although many other calendars are still used for local or religious purposes.

So, what was Clavius' contribution to the Gregorian reform of the calendar? Clavius was only in his early 30's when he was appointed to the commission. Yet he was the recognized local Roman expert on astronomy and mathematics, and also a member of the clergy on whom the Vatican could rely. Clavius was well-respected and eventually earned the epithet "the Euclid of his century." He had a conservative, traditional outlook, and was a staunch defender of the geocentric Ptolemaic system just as the revolutionary Copernican heliocentric system was emerging. Nevertheless, Clavius was a true scientist in the modern sense of the word and had the greatest respect for astronomical observations. In fact, under his supervision some of Galileo's telescopic discoveries were first confirmed. Clavius had the good political sense to steer the calendar reforms away from any dependence on the conflicting astronomical models of the day and instead used average values for such quantities as the length of the tropical year and the lunar month on which everyone could agree. As we have seen, the required leap year adjustment was not difficult to discern. Much more difficult was determining an accurate algorithm to calculate the date of Easter that stayed close to astronomical observations. Here Clavius made significant contributions. He was well aware that a system based on observation would be more accurate, but soon came to realize the increased technical difficulties would be out of proportion with the level of exactness gained. Finally, Clavius was a tireless advocate for promoting the adoption of the new calendar. He wrote six treatises in its defense, including the influential *Novi calendarii Romani apologia, adversus Michaelem Maestlinum*, (Rome, 1588) and the 600-page *Romani calendarii a Gregorio XIII restituti explicatio* (Rome, 1603). It can be confidently stated that the ultimate victory of the Gregorian reform was chiefly due to his efforts.

Modern efforts at calendar reform

The only significance of the Gregorian reform from a civil point of view is that it improved the method for inserting leap days into the calendar. Our modern calendar still has many peculiar features that have accumulated through the ages that serve no particular purpose. For example, the divisions of the year into months, quarters, or half years, are not equal, making it difficult to compare any activity or statistics of one period with another. Moreover, the days of the week constantly shift from one calendar year to the next. If this year begins on a Sunday, the next year will begin on a Monday, or Tuesday in a leap year. This shifting requires constant recalculation of schedules from year to year—we are all aware of the effort that goes into constructing a new academic calendar every year. Various methods have been proposed to solve these problems since the 19th century.

- The French Revolutionary Calendar had 12 months of 30 days divided into 3 decades. Five or six extra days were added each year. It was adopted for about 12 years until Napoleon restored the Gregorian calendar in 1806.
- In 1834 Abbe Marco Mastrofini proposed the *World Calendar*, also known as the *Universal Calendar*, consisting of 12 months with equal quarters. The months of a quarter had 31, 30, and 30 days. An extra *nameless day*—falling outside the usual sequence of the seven-day week and celebrated as a holiday—would occur at the end of the year, and the end of June in leap years. The *World Calendar* is exactly the same every year, simplifying scheduling. Each month/quarter/half year has the same number of work days.
- In 1849 Auguste Comte proposed the *International Fixed Calender* consisting of 13 months of 28 days. An extra month called *Sol* is added between June and July, and extra nameless days are added as in the *World Calendar*. This calendar was promoted by George Eastman (of Eastman Kodak fame) in the 1920's.
- The League of Nations before World War I and the United Nations after World War II considered various proposals to reform the calendar but no action was taken and no interest in reforming the calendar has been shown since that time.

There seems to be strong resistance to reforming the calendar, both for religious reasons—due to the disruption of holy days, or the Sabbath—and for civil reasons—due to the disruption of traditional dates for holidays. The introduction of a nameless day would disrupt the seven-day week, a deeply ingrained pattern many are reluctant to alter. This resistance is typified in the following passage from the Second Vatican Council document *On Sacred Liturgy* (December 4, 1963): "The sacred Council declares that it is not opposed to proposals intended to introduce a perpetual calendar into civil society. But among the various systems which have been mooted for establishing a perpetual calendar and introducing it into civil society, the Church raises no objection provided that the week of seven days with Sunday is preserved and safeguarded, without intercalating any day outside of the week, so that the succession of weeks is preserved intact—unless very serious reasons are forthcoming, of which the Holy See will be the judge."

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CHAPTER 3

What's so great about mathematics? Some Renaissance responses

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What's so great about mathematics?

Why is this question asked so often in the sixteenth century?

- Humanist reform of university curriculum
  - Mathematics theoretically part of medieval arts curriculum, but in serious decline in Oxford and Paris
  - Humanist recovery of ancient mathematical texts
  - But ambivalence about the subject itself

Utility

Utility central to humanist critique of the university curriculum.

- Juan Luis Vives: what is university logic *for*?
  - 'No non-man does not possibly not run'
  - 'Nothing and no one bite each other in a bag'
  - 'Surely it is self-evident that techniques which have to do with speech do not concern themselves with the delirious and foolish fictions of these men, but rather with things that are familiar to all the speakers of that language.'
- What, then, is mathematics for? At first sight, either:
  - a scholastic game much like terminist logic, pursued for its own sake, *or*
  - a practical art used by navigators, merchants and architects.
- In either case it did not recommend itself to the classes who sent their children to the universities.

Utility—Proclus

A promising ancient answer: Proclus. Proclus (412–85), *Commentary on the First Book of Euclid's Elements*

- *Editio princeps* of Greek text: Basel, 1533 (by Simon Grynaeus) – together with *editio princeps* of Euclid's *Elements*.

- First Latin translation: Padua, 1560 (by Francesco Barozzi).
- Mathematics in middle between intelligibles and the material world.
- Utility was to lead the mind upwards towards the Forms. Not just useful, but *essential* propaedeutic to philosophy.
- History of mathematics carefully crafted as progression from the more material to the more abstract and 'philosophical', culminating with the 'Platonist' Euclid.
- Popular in part because:
  - practically the only extended treatment of the philosophy of mathematics from antiquity, and
  - seemed to offer the best chance for integrating the sciences into a humanist curriculum.

Some early followers of Proclus.
- Girolamo Cardano, *Encomium geometriae* (delivered in Milan, 1535).
- Melanchthon, preface to Johannes Vögelin, *Elementale geometricum* (1536).

Petrus Ramus

The *Prooemium*.
- Written during (and in response to) several crises:
  - Conversion to Reformed religion, and outbreak of wars of religion.
  - Disillusion with mathematics as 'natural' art. (Mathematics is not easy).
  - Appointment of Jacques Charpentier as Regius Professor of Mathematics.
- *Prooemium* is fiercely anti-Platonic & anti-Euclidean; seeks to be better Aristotelian than Charpentier.
- Physics/natural philosophy is (or should be) 'completely mathematical'. Model of Pythagoras. Stick to beat Aristotle/Aristotelians?
- The 'walk through Paris'—ubiquity of mathematics among the unlearned. (Also in speech to University Senate against Charpentier).

Jacques Charpentier

Admonitio ad Thessalum (1567)

- Mockery of the walk through Paris
- Bewilderment at Ramus's championing of Pythagoras.
- Ramus has started asserting in his lectures that mathematics constitutes the essence of things, prior to substances.

Henry Savile (1549–1622)

Oxford lectures on *Almagest* (1570).

- Highly indebted to Ramus's *Prooemium* ...
- ... but also critical of Ramus's practical answer to the question of utility.
- Proclus is central—hyper-Platonism.
- Applicability of mathematics in a craft/merchant context is *per se* a reason for his students to hold it in contempt.
- Mathematics is an indicator and source of moral goodness: Aristippus.
- Professorships (founded 1619) will be carbon copies of Ramus's advice in *Prooemium*.

Optics

The utility of an applied science.

- A bigger problem for most academic writers on the utility of mathematics.
- Examples of artists and architects not generally appropriate in university setting.

Jean Pena. Preface to translation of Euclid's *Optics* and *Catoptrics* (1557). Applications of optics:

- Astronomy and natural philosophy of the heavens.
- Detecting the impostures of witches.
- Despite coming from Ramus's circle, no mention of quotidian uses of optics (in particular, spectacles).

Utility of optics

Johannes Alsted, *Elementale mathematicum* (1611).

- Optics receives least room ...
- ... but is the only science to merit its own 'encomium'.

Al-kindian/Baconian tradition.

- John Dee, *Propaedeumata aphoristica* (1558/1568)
- Henry Briggs, inaugural lecture on mathematics (Cambridge, 1588)

Situation changes post-1610/11 (Galileo and Kepler). For example, Brian Twynne.

Dignity of mathematics

- Even if mathematics is *useful* (in some acceptable way) isn't is *undignified*?
- Most common answer: antiquity ⇒ dignity.

Antiquity of mathematics.

- Story from Josephus, *Jewish antiquities*
- Repeated in Eusebius, *Praeparatio evangelii*
- In Ramus's version in *Prooemium*, mathematics predates all other arts and sciences.

Clavius on utility and dignity

Preface to commentary on Sacrobosco.

- First authority cited: Proclus. But not overwhelming influence.
- Antiquity of mathematics: studies of the patriarchs.
- Emphasises *continuity* of present practitioners with antiquity: patriarchs qualitatively the same.
- Largely conventional on *utility*: philosophy, theology, poets—and church/calendar, of course!
- Columbus's use of eclipse.

Preface to Euclid.

- Account is incompatible with that of Sacrobosco edition. No interest in patriarchs, but emphasis on (quasi-)historical origins.
- What begins as subject of vital importance, devolves into merely conventional.

CHAPTER 4

The Clavius Project: Putting Clavius' *Opera Mathematica* Online

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Outline

I would like to talk about four aspects of putting Clavius' *Opera Mathematica* online.

- Critical decisions for the project
- Review of our "mission" and what we learned
- Demonstration of the Clavius Web site
- Publicizing the project and what to do next

Cast of characters

There were a number of folks who were helpful during the project. I'll mention specifics during the talk, but let me give you a general sense of their contributions.

Fr. Joseph MacDonnell, S.J., Clavius Group—Fr. MacDonnell generated the initial interest in this project, arranged for the Clavius Group to fund part of it, commissioned John Blanton, St. John Fisher College, Rochester, New York, to translate the preface (or dedication), and provided advice as we made critical decisions. He and I communicated mainly via e-mail, and I regret not ever having met him in person.

Professor Dennis Snow, Mathematics Department—Dennis was my main point of contact with both the Clavius Group and Notre Dame's mathematics department. He kept us from building a "Cadillac," and focused us on what the Clavius Group (and other research mathematicians) might want.

Parker Ladwig, Mathematics Librarian—I have been the overall coordinator for the project since it began in 2001.

Eric Morgan/Rob Fox, Digital Access and Information Architecture Department, University Libraries—Eric was helpful in providing technical advice about the digitization, about the way to present the images once digitized, and for coordinating the work of his department. Rob was largely responsible for creating the Web site, converting the digitized TIFF images into JPG files, and adding features requested by Dennis. The project could not have been completed without Rob's technical expertise.

Liz Dube/Dorothy Snyder, Preservation Department, University Libraries—Liz was helpful in providing technical advice about the digitization, for arranging the digitization itself with Preservation Resources, for coordinating the funding for the project, and for coordinating the work of her department. Dorothy arranged to get microfilm from St. Louis University, inspected it and made recommendations, and processed the work from Preservation Resources. Dorothy's advice was critical early in the project.

Lou Jordan/Joe Ross, Special Collections, University Libraries—Lou was helpful in providing advice on digitization projects from a rare books perspective and for providing general support. Joe helped maintain interest in the project and arranged for today's special display of Notre Dame's collection of Clavius publications.

Gay Dannelly/Nigel Butterwick, University Libraries—Gay and Nigel provided important administrative support from the Libraries, arranged for funding as necessary, and kept an eye on the project's progress.

Critical decisions for the project

Now that we've talked about the key team members, let's talk about some of the critical decisions that had to be made during the project. I've organized them into five areas.

(1) Why do the project and who will pay for it?
(2) Do we digitize the print or some derivative work?
(3) What is the appropriate quality for the images?
(4) How do we organize the images?
(5) How do we make it available on the Web?

1. Why do the project, and who will pay for it? During the 2000/2001 academic year, the mathematics library was given one-time funding to purchase collected works of mathematicians. I put together a list of collected works available that weren't owned by Notre Dame and gave that list to the department's library committee. One of the works on the list was Clavius' *Opera Mathematica*.

I checked to see if it could be purchased, but since it was published in 1612, I didn't expect to find it available. I did not. I'd heard, however, of the Clavius Group, and thought I'd contact a local member, Professor Dennis Snow. Dennis thought that even if we couldn't purchase the *Opera Mathematica*, we might be able to get a digital version that we could print out and put on our shelves. He also said that he would contact someone from the Clavius Group to get another opinion.

In March, 2001, Fr. MacDonnell e-mailed Dennis back about the importance of the project. "It is an odd story. Many have tried to get their hands on Clavius' works (including myself) for some years ...." Thus, the digitization project began.

The first problem was to sort out funding arrangements. In July, 2001, Dennis e-mailed, "The Clavius Group is willing to contribute $1,500 to the project if Notre Dame will pay the remaining cost. Everyone here agrees that converting the microfiche to CD-ROM is the way the project should be done. (There was no real support for producing a facsimile copy.) We hope that a translated index would be part of the package with links to the appropriate pages." This e-mail was essentially the mission statement for the project.

In April, 2002, I summarized the reasons for the project as follows:

(1) Funding is in place (Clavius Group $1,500; ND the rest)
(2) Printing from microform reader would be cumbersome (I had discovered that Notre Dame did have a microfiche version of the *Opera Mathematica*)
(3) Opportunity for ND to learn
(4) Print copies could be made from the images
(5) Could encourage a critical edition

2. Do we digitize the print or some derivative work? The next critical decision was whether to digitize an existing print version or some derivative work (microfilm or microfiche, most likely).

My preference was to digitize the print. When I contacted the U.S. Naval Observatory, Fordham, and Harvard in the spring, 2001, however, each told me that their copy was in such poor condition that they would not loan the copy nor would they recommend making a digital version from it.

Arranging to have it digitized at another university for Notre Dame also didn't make sense, so in the summer and fall, 2001, Lou Jordan, Liz Dube, Eric Morgan, and I began to think about using the microfiche version available at Notre Dame. It looked like there were going to be problems with copyright and with the quality of the images, so we began to think about the possibility of using microfilm.

During the remainder of the 2001–2002 academic year, I arranged to borrow a microfilm version from St. Louis University, and Dorothy Snyder inspected it and suggested that it would be good enough for the purposes of our project. After discussing Dorothy's recommendation with Lou, Liz, Eric, and Dennis, I e-mailed Fr. MacDonnell in June. I said that the microfilm was "high enough quality for the purposes intended"; Fr. MacDonnell wrote back, "So go ahead."

3. What is the appropriate quality for the images? After we had decided to use microfilm for the digitization, we then had to agree on the technical specifications for the digital images.

In the summer, 2001, Lou had e-mailed, "I thought you would have 600 dpi TIFFs produced from microfilm. These would be stored on CD-ROM's .... Then copies of the TIFFs could be reduced in size and converted to PDFs or any other file format you may wish to use (another popular option is JPEGs)." His recommendation was particularly prescient, as you'll see.

After discussing it more among the library team members, Liz wrote "I might suggest the following test: All bitonal TIFF scans, 10 pages each of: 300 dpi, 400 dpi, 500 dpi and 600 dpi." Liz arranged this test with Preservation Resources, the company that actually did the digitization for the library.

We looked at the test results and found that bitonal images (black and white) did not work well because some pages were darker than others (and thus, illegible). We then arranged for grayscale test images and reviewed the results with the team.

In the fall, 2002, Liz told Preservation Resources to digitize the microfilm into 400 dpi grayscale TIFF images. Now the digitization would begin in earnest.

4. How do we organize the images? In February 2003, Notre Dame received a box of 94 CD-ROMS; the TIFF image for each page of the *Opera Mathematica* was about 1.5 MB large. By May, 2003, the Mathematics Library's student workers had completed an inspection of each image (for quality and missing pages). I then asked Fr. MacDonnell to pay the Clavius Group's part of the project cost.

During the inspection process, we began to think more about how to organize the images. In May, I tried making a PDF file from several TIFF images, but it was a cumbersome process. In June, 2003, Eric suggested converting the TIFF images into JPG files and "integrating them into a *page turning* program of our own design or written by another university library."

During the 2003–2004 academic year, we investigated several possibilities: METS tools (New York University), DigiTool, DLXS (University of Michigan), and Greenstone (from New Zealand). In March, 2003, we decided to use Greenstone, an open source page-turner software program.

In the fall, 2003, I learned of another digitization project that nearly put an end to ours. The Garden of Archimedes project, Florence, Italy, had created a PDF version of Clavius' *Opera Mathematica* and was selling the CD-ROM for about $150. Notre Dame's project was put on hold until the CD-ROM could be examined.

In January, 2004, after discussing it with Dennis, we decided to proceed. The Clavius Group's intention had been to make the *Opera Mathematica* freely available and now that we had already made and paid for the images, he thought that it made sense to continue.

For the rest of 2004, Eric and his department tried to get the Greenstone software to work (they were also working on a major overhaul for most of the University Libraries' Web sites). By January, 2005, we decided to give up and, with Rob Fox's help, to design our own page-turner software.

In December 2004 we had already decided to convert the TIFF images into three JPG images—one for viewing, one for "zooming in," and one for printing.

5. How do we make it available on the Web? By the end of January, Rob had developed a draft of the Web page-turner. He made it available to the entire team, including Dennis and Fr. MacDonnell. After the team had approved it, Rob and I began to convert each TIFF image to three JPG images using a utility that he also developed.

By March, all of the images were available on the Web site and an "about" page was created. Dennis asked Rob and Eric if they could make a CD-ROM version available for distribution to members of the Clavius Group.

By June, the details about cataloging the Web site, copyright, and hyperlinks to other Web sites had been finalized; the CD-ROM version was also completed. Essentially, the digitization project was finished.

Review of our "mission" and what we learned

Those were the five critical decisions during the project. Let's now review the "mission" articulated by Dennis and then talk about what we learned.

In July, 2001, Dennis had written, "... Everyone here agrees that converting the microfiche to CD-ROM is the way the project should be done. (There was no real support for producing a facsimile copy.) We hope that a translated index would be part of the package with links to the appropriate pages."

In April, 2003, I had written:

(1) Funding is in place (Clavius Group $1,500; ND the rest)
(2) Printing from microform reader would be cumbersome
(3) Opportunity for ND to learn
(4) Print copies could be made from the images

(5) Could encourage a critical edition.

We did convert a microfilm version to digital images available on the Web and on a CD-ROM. There is the capability of printing individual images. Translation work was begun, and indices originally published in the *Opera Mathematica* are highlighted on the Web site. The project was funded jointly by the Clavius Group and the University Libraries of Notre Dame. It has been an opportunity for Notre Dame to learn about the digitization process. Finally, depending on how the project is publicized, there is some hope of generating interest in Clavius and his work among researchers.

Demo of the web site

Here followed a demonstration of the Web site. See `http://mathematics.library.nd.edu/clavius/` and `http://mathematics.library.nd.edu/clavius/about/about_page.html`.

Publicizing the project and what to do next

Clavius' *Opera Mathematica* is now available on the Web (and on CD-ROM). How should we publicize the project and what should be done next?

There have been two marketing efforts already:

- February 7, 2005—"Team to launch rare mathematics volume online" published in *ND Works*, vol.2, no.10.
- July 2005—this Clavius symposium to present the Web site and discuss improvements/next steps.
- Dennis and I are thinking about publishing Fr. McDonnell's biographical sketch of Clavius and/or publishing the "proceedings" of this symposium (and making them available on the Web).
- Several next steps have also been identified: Commission more translation work, first the preface to Clavius' *Commentary on Euclid*, the "Prolegomena to the Mathematical Disciplines," and then the preface to his*Commentary on Theodosius' Spheres*.
- Digitize the pages missing from the microfilm. There are about 30 pages missing—a print edition needs to be found, and those pages need to be digitized.
- If there are no other suggestions for how to publicize the Clavius Web site and what to do next, I'll answer in questions you might have about the project.

Thank you for your attention, and thank you for giving me the opportunity to make Clavius' work more accessible.